

## Automatic Singlet Peak Analysis of Hpnal YsisOf Hp (Ge) $\gamma$ -Ray Spectrum



Aziz H. Fattah, \*Ali H. Ahmed and Mariwan A. Rasheed

Department of Physics / College of Science / Sulaimani University, Kurdistan Region /Iraq.

\*Department of Physics / College of Science / Salahaddin University, Kurdistan Region /Iraq.

### Abstract

A computational program is applied for analyzing of  $\gamma$ -ray spectra which obtained with high-pure germanium, Hp(Ge), spectrometers in Erbil region. The program concerns the individual peak identification by defining the position and region of the peak that is significantly above the local spectral background. Each peak region, that is represented by a gaussian function underlying a linear background, is fitted by following Marquardt searching method to get the best peak parameters. By using the coded program, relative intensities and energies have been calculated for each singlet peaks.

**keywords:-** Nuclear physics – detection of nuclear radiation-analysis of gamma ray spectrum

### Introduction

This paper outlines a computer program, which has been developed as an aid in analyzing of a natural  $\gamma$ -ray spectrum that recorded with Hp(Ge) detectors. The program is designed to locate automatically all the significant individual peaks and their regions in the read-in spectrum. It is written to include Marquardt method for fitting then determining accurately the peak components for computing  $\gamma$ -ray energies and intensities. Every spectrum analysis by hand or with a computer program may produce peak shape model that depends results through the algorithms used. The peak shapes in typical  $\gamma$ -ray spectrum are normally analyzed on the assumption that a peak shape is represented by a Gaussian distribution. In practice, however the peaks usually possess an element of asymmetry and may displays tailing [1].

The method of analyzing complex peaks occurring in  $\gamma$ -ray pulse high distributions is obtained with multichannel pulse height spectrometers. The utilized data were taken from radioactive natural samples

and recorded with a high-pure germanium detector, Hp(Ge)in Erbil[2].

### Peak representation

The results of radiation measurements are, in most cases, expressed as the number of recorded counts. These counts indicate that particles have interacted with the detector and produced a pulses that has been statistically recorded.

The physical and statistical phenomena determining the response of the semiconductor detector to a monoenergetic  $\gamma$ -ray is quite complex and the accurate fundamental calculation of peak shape for the purpose of spectrum analysis is then quite difficult [3]. For this reason and because the peak shape is sensitive to small variation in the experimental parameters, it is described to determine mathematical representation directly from the measured data.

A wide variety peak-fitting and different representation of peak shape have been proposed in the past. However, there is no

universally accepted  $\gamma$ -ray peak-fitting and representations that can claim to be reliable and applicable in all cases [4]. The majority of peak The effect of the tailing can be described by a further gaussian which has the of peak representations are Gaussian functions ridding on a continuum background. The separation of the data into photo-peaks and background is of paramount if good estimating of intensities are to be obtained, as followed by ref.[5]. Since much of the background arises from Compton effects in the detectors, thus, the background in any local region is related to the energies and intensities of photopeaks [6]. Another gaussian function is proposed to define tailing effect in the peak boundaries.

A typical  $\gamma$ -ray full energy peak having a very low background was carefully studied for determining an analytical expression that would very closely represent its shape [7]. The peak can be described by Gaussian functions and the background may be approximated by linear function sources within the peak interval. Thus, in such an interval the number of counts as continuous function of the channel number (x) is given by:-

$$f(x) = h \exp\left(-\frac{(x-x_c)^2}{2\sigma^2}\right) + a + bx \dots\dots 2.1$$

The first term in the equation is a gaussian of height (h), centroid (xc) and standard deviation ( $\sigma$ ). While the second term is a linear background that joins the main gaussian and the combined function with its derivative continuous, shown in the

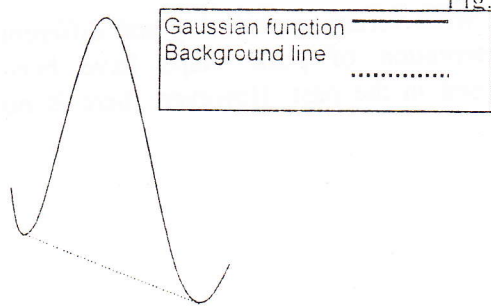


Fig.(1): A linear background underlying the gamma-ray photo peak.

parameters high ( $h_T$ ), centroid ( $x_{cT}$ ) and standard deviation ( $\sigma_T$ ). So it can be represented by:

$$f_T(x) = h_T \exp\left(-\frac{(x-x_{cT})^2}{2\sigma_T^2}\right) \dots\dots 2.2$$

that is not fitted; but it will be added automatically to eq.(2.1) if needed, to remove asymmetry of the peak.

### Mathematics and options in the program

#### Program's description

The present study reports a computer program, named CURFIT, which follows Marquaedt method in the fitting technique [8] and developed to determine requires quantities for  $\gamma$ -ray spectrum analysis. The program was coded to analyze the spectrum automatically, provided that the following input quantities are stored in the open files or as direct inputs:-

- 1-Entire spectrum includes the peak counts in channel ranges (1-4095).
- 2-Energies from the library of isotope descriptions are read in that are arranged in a list to distinguish the peaks of the spectrum.
- 3-Calibration constants of energy and efficiency.
- 4-The parameter step sizes that are required in the fitting process.

The subroutines SMOOTH, FOURIRI, REALFT, GSER and GCF and subprograms (ERF, GAMMP and GAMMLN) were developed in the program [9]. While the routines CURFIT, FDERIV and MATINV) and programs FUNCTN were modified by Bevington [10] to the equivalent one.

A short description of the main program and the various subroutines and functions is

given below:-

MAIN PROGRAM: After reading and inputting the required quantities. SMOOTH routine is called and the individual regions in the spectrum may be distinguished, then the routine FIND is called.

In the program the following subroutines and subprograms are called:

**SMOOTH:** It is called in the main program to define the smooth data, that is calls routines REALFT and FOURIERI to do discrete Fourier transformations with input spectrum.

**FIND:** Its called from the main program, for searching the peaks then finding the channel boundaries and the initial values of parameters in the distinguished smoothed region.

**CURFIT:** Its called from FIND routine for fitting the peak data by using marquardt method after initializing parameters.

**FDERIV:** This routine calculates first derivatives of the gaussian function with respect to the parameters (amplitude, centroid and width), it is called by CURFIT.

**FUNCTN:** It is the program that is used from CURFIT routine for calculating the value of the fitting function for the peaks at each point of the spectrum.

**INVI:** It determines the inverse of the curvature matrix (Hessian matrix), it is called from CURFIT.

**ERF:** It is called from subroutine FIND to use GAMMP (that is also uses GCF and GAMMLN) for calculating error functions to evaluate the tailing contributions.

### Smoothing and peak searching

The peak-search routine uses smoothed spectrum obtained so that the process of data smoothing based on the inspection of the frequencies in the Fouriers transformed spectrum [11,12].

To perform an automatic analysis of  $\gamma$ -ray peaks one must provide the program with an algorithm like data smoothing that attempts to define the ranges of channels (it divides the spectrum into many regions) in the spectrum which lie significantly above the background to simplify the process of peak fitting, see Fig.(2).

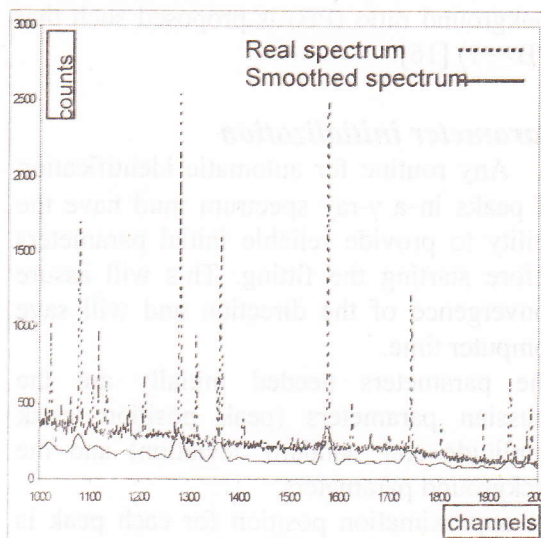


Fig.(2): A part of Hp(Ge) spectrum with its associated smoothed spectrum among the channel ranges (1000-2000).

Since any  $\gamma$ -ray photo peak is superimposed on a fluctuation background, another important role of data smoothing is to avoid the spurious (noise peaks) due to statistical fluctuations [13].

The automatic identifications of peaks is usually based on the behavior of the first differences [14].

The process of peak finding is done by calling Subroutine (FIND), so it searches for peaks by locating adjacent channels such that the derivative (the differences) in the left-hand number (left side of the peak) of the pair of channels is positive, i.e., when the peak begins to rise from the bottom up to the top of the peak and the successive counts satisfy condition  $(y_j > y_{j-1})$ , ( $j$  and  $j-1$  are successive channels) but; in the right-hand number (right side of the peak) is negative or zero, i.e., where the peak begins to go down from top to the bottom of the peak and the successive counts satisfy  $(y_j < y_{j-1})$  [15]. As a result of the searching process, the boundaries for each peak (starting channel and final channel) with their corresponding counts are determined.

To neglect the weak peaks (noise peaks) through the peak searching, peak-to-

background ratio (PB) is proposed such that (PB>=1) [16].

**Parameter initialization**

Any routine for automatic identification of peaks in a γ-ray spectrum mud have the ability to provide reliable initial parameters before starting the fitting. This will assure convergence of the direction and will save computer time.

The parameters needed initially are the gaussian parameters (peak position, peak amplitude and standard deviation) and the background parameters.

An approximation position for each peak is evaluated from three values of closest counts y<sub>k</sub> to the peak position.

$$P = j_c + \frac{y_{j_c+1} - y_{j_c-1}}{2R} \dots\dots 3.3.1$$

where j<sub>c</sub> is the peak position and (R) takes the values:

$$R = y_{j_c} - y_{j_c+1} \text{ if } y_{j_c+1} > y_{j_c-1} \dots\dots 3.3.2$$

$$R = y_{j_c} - y_{j_c-1} \text{ if } y_{j_c-1} \leq y_{j_c+1} \dots\dots 3.3.3$$

Peak position determined in this way differ from the exact values by less than 0.15 channels in most cases.

Depending on eq.(2.1), starting value of the standard deviation (σ) can be evaluated from the following formula [17]:

$$\sigma = \frac{|j - j_c|}{\sqrt{1 - \frac{(j - j_c)}{r}}} \dots\dots 3.3.4$$

Where (j) represent the channel numbers (j<sub>c</sub> is the centroid channel) and the value of (r) is calculated by the equation:-

$$r = \frac{f'(j)}{f''(j)} = \frac{\sigma^2(j - j_c)}{\sigma^2 - (j - j_c)^2} \dots\dots 3.3.5$$

and f'(j) and f''(j) are the first and second derivatives respectively, which are determined numerically by using the differences. The background parameters (a) and (b) in eq.(2.1) are found from the slope of

the background line for first and the last channel [18], or

$$a = \frac{y_j(j_i - j_f) - x_i(y_i - y_f)}{(j_i - j_f)} \dots\dots 3.3.6$$

and

$$b = \frac{(y_i - y_f)}{(j_i - j_f)} \dots\dots 3.3.7$$

such that (j<sub>i</sub>, and j<sub>f</sub>) are symbolized to the first and the last channels, while; y<sub>i</sub> and y<sub>f</sub> are their respective counts for each peak.

**Peak fitting**

In this analysis it is assumed that the data points in any portion of the experimental spectrum (peak) can be represented by a function y(j,p<sub>j</sub>) which depends on a set parameters (p<sub>k</sub>). Then the Marquardt method, which combines the best feature of the gradient search method and linearization [8], is followed to fit the data.

The purpose of the fit is to find the values of the parameters, which minimize the sum of the squares (reduced chi-squares) of the data from the fitted function, that is for n-data points which are given by:

$$x^2 = \frac{1}{v} \sum_j^n w_j (y_j - y(j, p_k))^2 \dots\dots 3.4.1$$

Where (v) is symbolized to degree of freedom that is the difference between number of data points (n) and the number of Gaussian parameters (m=5) and y<sub>j</sub> are the experimental data pints (observed points or counts). While; w<sub>j</sub> are the weights associated with y<sub>j</sub> that can be evaluated from:

$$w_j = \frac{1}{y_j} \dots\dots 3.4.2$$

But y(j) are symbolized to the estimated data points calculated from the function used to represent the data that is approximated to a Gaussian ridding on a linear background, or eq.(2.1) is rewritten as:

$$y(j) = p_1 \exp\left(-\frac{(y - p_2)^2}{2p_3^2}\right) + p_4 + p_5 j \dots 3.4.3$$

Since  $y(j)$  is a function which is nonlinearly dependent of the parameters, ( $p_1, p_2$  and  $p_3$ ) for the Gaussian and ( $p_4$  and  $p_5$ ) for the background, it makes ( $x^2$ ) a nonlinear function also. Therefore, to apply linear least square in eq.(3.4.1),  $y(j)$  in eq.(3.4.3) must be linearized by taking its linear expansion (Taylor series) for number of parameters ( $m=5$ ), i.e., by neglecting all terms above the first derivative so it becomes:

$$y(j) = y_0(j) + \sum_k^m \frac{\partial y_0}{\partial p_k} \delta p_k \quad \dots\dots\dots 3.4.4$$

that the function  $y(j)$  becomes a linearized in the parameter increments and the method of linear least squared is used in the fitting, such that eq.(3.4.1) is changed to:

$$x^2 = \sum_j^n w_j (y_j - y_0(j) - \sum_{k=1}^m \frac{\partial y_0}{\partial p_k} \delta p_k)^2 \dots\dots\dots 3.4.5$$

In order that the parameters take the best values  $x^2$  (which is a scale of difference between the observed data and estimated data) must have local minimum values [18]. Fixing the starting values for the fitting procedure required some experience, because as a whole, several relative minima of the reduced chi-square exist besides the absolute minimum [20]. Thus, a necessary and sufficient condition for the chi-square in the above equation to be a local minimum (as a function of  $p_1$ ), is

$$\frac{dx^2}{d\delta p_1} = 0 \quad \dots\dots\dots 3.4.6$$

hold simultaneously for all values of  $k$ , then it requires:-

$$0 = -2 \sum_j^n w_j [y_j - y_0(j) - \sum_{k=1}^m \frac{\partial y_0}{\partial p_k} \delta p_k] \frac{\partial y_0}{\partial p_l} \dots\dots\dots 3.4.7$$

To get a new step length in parameters ( $\delta p$ ), one must solve a system of  $m$ -simultaneous equations that arranged as below:

$$\beta_l = \sum_{k=1}^m \delta p_k \alpha_{kl} \quad \dots\dots\dots 3.4.8$$

where the matrix elements  $\beta_k$  and  $\alpha_{kl}$  (curvature matrix) are calculated from:-

$$\left. \begin{aligned} \beta_k &= \sum_j^n w_j (y_j - y_0) \frac{\partial y_0}{\partial p_k} \\ \alpha_{kl} &= \sum_j^n w_j \frac{\partial y_0}{\partial p_k} \frac{\partial y_0}{\partial p_l} \end{aligned} \right\} \dots\dots\dots 3.4.9$$

The first derivative are assumed to be calculated at starting point ( $y_0$ ) from:-

$$\frac{\partial y_0}{\partial p_k} = \frac{[y_0(j, p_k + \Delta p_k) - y_0(j, p_k - \Delta p_k)]}{2\Delta p_k} \quad 3.4.10$$

by taking into account that  $\Delta p_k$  are the step sizes in the local minimum point that take the values (1, 0.1 and 0.01) for the Gaussian parameters (height, centroid and width) respectively and (0.05 and 0.01) for background parameters for (a and b) respectively, see eq.(2.1).

Following Marquardt method in fitting, it requires to increase the diagonal elements of curvature matrix ( $\alpha$ ) by a factor  $\lambda$  which controls the interpolation of the algorithm between the two extremes (local minimum points). Thus eq.(3.4.8) becomes:

$$\beta_l = \sum_{k=1}^m \delta p_k \alpha'_{kl} \quad \dots\dots\dots 3.4.11$$

where the new curvature matrix ( $\alpha'$ ) is given by:

$$\alpha'_{kl} = \begin{cases} \alpha_{kl}(1 + \lambda) & \text{for } k = l \\ \alpha_{kl} & \text{for } k \neq l \end{cases} \dots\dots\dots 3.4.12$$

The solution of the parameter increments (step length,  $\delta p$ ) is calculate from:

$$\delta p_l = \sum_{k=1}^m \beta_k \alpha'^{-1}_{lk} \quad \dots\dots\dots 3.4.13$$

Thus, for  $k^{\text{th}}$  iteration the new values of the parameters are determined from:

$$p_l^{k+1} = p_l^k + \delta p_l^k \quad \dots\dots\dots 3.4.14$$

and the error estimation in the parameters ( $\sigma p_k$ ) are evaluated from the inverse of the diagonal elements of  $\alpha'$  [9], or

$$\sigma(p_k) = \alpha'^{-1}_{kk} \quad \dots\dots\dots 3.4.15$$

Finally, the step recipes of the coded Marquardt method can be abbreviated as:

Compute  $x^2(p)$ .

Start initially with  $\lambda=0.001$ .

Compute  $\delta p$  and  $x^2(p+\delta p)$  with this choice of  $\lambda$ .

If  $x^2(p+\delta p) > x^2(p)$ , decrease  $\lambda$  by a factor of (10) then repeat step (3).

If  $x^2(p+\delta p) < x^2(p)$ , decrease  $\lambda$  by a factor of (10), consider  $(p'=p+\delta p)$  to be a new starting point, then return to step (3) substituting  $p'$  for  $p$ .

**Peak analysis**

After fitting process to estimate best values of the parameters and their uncertainties, for peak analysis, these steps are followed:

A) By subtracting underlying background function from the curve of eq.(2.1), the net Gaussian is formed, for which the peak area (A) can be computed by the following analytical method using the Gaussian parameters, height (h) and standard deviation ( $\sigma$ ) [19]:

$A = \sqrt{2\pi}h\sigma$  ..... 4.1

and determining its uncertainty  $\sigma(A)$  from:

$\sigma(A) = 165 \times \frac{\sqrt{A + 2 \times SB}}{A}$  ..... 4.2

where (SB) is the summation of the count on the background.

B) By transforming peak position to the energy from energy calibration constants, then by transforming the peak energy to the efficiency from efficiency calibration constants, the peak energy (E) and efficiency ( $E_f$ ) can be evaluated respectively:-

$E = w_1 + w_2 \times x + w_3 + x^2$  ..... 4.3

its uncertainty  $\sigma(E)$  is given by:-

$\sigma(E) = (w_2 + 2 \times w_1 \times x)$  ..... 4.4

where  $w_1$ ,  $w_2$  and  $w_3$  are energy calibration constants, while;

$E_f = z_1 E^{z_2}$  ..... 4.5

where  $z_1$  and  $z_2$  are efficiency calibration constants.

C) Since the intensity of the  $\gamma$ -ray lines is characterized by the corresponding peak area, then the intensity of a particular line is the ratio of the peak area to the efficiency [18]:

$I = \frac{A}{E_f}$  ..... 4.6

and

$\sigma(I) = \frac{\sigma(A)}{E_f}$  ..... 4.7

The relative intensity ( $R_i$ ), where intensities are normalized to (100), can be written as:-

$R_i = \frac{I}{I_s} \times 100$  ..... 4.8

where ( $I_s$ ) represents intensity of the standard line. Then its uncertainty is:

$\sigma(R_i) = \frac{\sigma(I)}{E_f}$  ..... 4.9

**Results and discussion**

After data smoothing, the process of peak identification and fitting were performed successfully for each singlet  $\gamma$ -ray line to determine a group of parameters that give the lines the best Gaussian shapes (of minimum chi-squared). In the fitting technique, getting the best Gaussian parameters is depended on the chi-squares (differences between the estimated and observed data points), if the statistics of the measured spectrum (weights) is not considerable. Arrival to those best parameters is depended on whenever their starting values may be chosen, such that for the quickly degree of this arrival, number of iterations (IT) is indicated (i.e., small IT means quickly arrival and large IT for slowly arrival), as observed in Table (1).

**Table (1): Some of the calculated results taken from the output of the run program.**

No.	Energy ( $E_\gamma$ ) in keV	Gaussian parameters			Peak area + % error	TP	PB	CH	IT
		Height (h)	position ( $x_0$ )	Width ( $\sigma$ )					
1	238.58± .004	17475.2±140.0	329.4± .006	.75± .006	32674.8± 1.4	0.001	6.1	15.04	9
2	258.83± .052	717.2± 59.6	358.0± .074	.78± .078	1404.5± 18.9	0.004	1.3	5.19	7
3	295.20± .004	19680.2±131.6	409.5± .005	.82± .005	40615.1± 1.1	0	10.3	0.23	8
4	338.31± .013	3160.6± 64.5	470.5± .018	.85± .020	6707.3± 4.0	0.002	3	0.71	6
5	351.92± .003	30910.4±156.4	489.8± .004	.86± .003	66328.6± 0.8	0	24.7	8.47	6
6	454.69± .158	214.7± 58.6	635.2± .223	1.22± .321	656.6± 23.8	0.05	1.1	0.02	64
7	562.74± .203	133.0± 34.1	788.2± .288	1.07± .404	355.1± 37.4	0.018	1.3	2.32	13
8	609.46± .003	18705.8±112.7	854.3± .005	.97± .004	45366.2± .9	0	56.7	10.4	9
9	703.02± .138	152.1± 27.4	936.7± .196	1.11± .283	422.4± 33.9	0.018	1.4	0.32	8
10	904.04± .123	106.5± 21.1	1271.2± .174	.82± .218	219.3± 44.1	0.002	1.4	0.22	10
11	911.09± .013	2270.1± 40.2	1281.2± .018	1.07± .017	6113.6± 2.9	0	9.8	2	7
12	1460.84±.010	3223.5± 41.1	2059.5± .014	1.29± .012	10397.4± 1.8	0	27.9	4.28	12
13	1620.61±.207	44.0± 8.0	2285.7± .293	1.48±.356	162.8± 38.7	0	1.8	0.76	7
14	1729.56±.036	340.1± 14.0	2440.0± .052	1.34± .049	1143.7± 6.7	0	10.1	1.44	5
15	1764.68±.015	1538.4± 26.2	2489.7± .021	1.45± .022	5602.4± 2.3	0.001	31.3	6.41	24
16	2213.05±.433	19.2± 11.6	3124.7± .613	1.65± .206	79.5± 35.3	0.029	1.7	0.58	10

In determining of relative intensities, the processions are generally limited by the ability to define and compute consistent peak areas. According to the listed values of the areas and peak-to- background ratio (PB) in Table (1), the peak area evaluation concerns strongly with the base line (background) height and the percentage error in the peak area goes up as (PB) decreases because the statistical fluctuations are more poorer. Consequently, the detection of the weak peaks (small PB) in the spectrum is complicated because of the appearance of fraudulent peaks which are nearly noise and the fitting of those peaks are very sensitive to the height of the background under the line as a result of the great area contribution of the background in the photo peak. The asymmetry of the peaks was removed by adding a tailing function to one of the peak sides (especially for the low energy side) such that the percentage area contribution of the tailing function (TP) must be restricted so that situates between (0-5%) of the total peak area, Table (1).

It is clear from the analysis that the

whole spectrum, as shown in Fig.(3), includes about (56) singlet  $\gamma$ -ray lines for (seven) radionuclides with energy ranges between (128-2614 keV) as shown in Table (2).

The emission rates (relative intensities), that obtained from the present work, are compared with the results of Nuclear data sheets (N.D.S.) [21,22,23,24,25].

The comparison in Table (2) shows that the present results (relative intensities and their corresponding uncertainties) are in agreement with those of (N.D.S.), specially for the strong lines, because the results of (N.D.S.) are obtained for the spectra of radionuclide sources not samples. In the spectrum the line with energy (1460.84 keV) from  $K^{40}$  nuclide is clearly observed and assigned qualitatively due to its non-serious radioactive origin.

It is worth mentioning that in the spectrum no attention was paid to peaks below (100 keV) since these were likely to correspond to X ray or  $\gamma$ -X interference, and because of the low (PB) ratio of these lines.

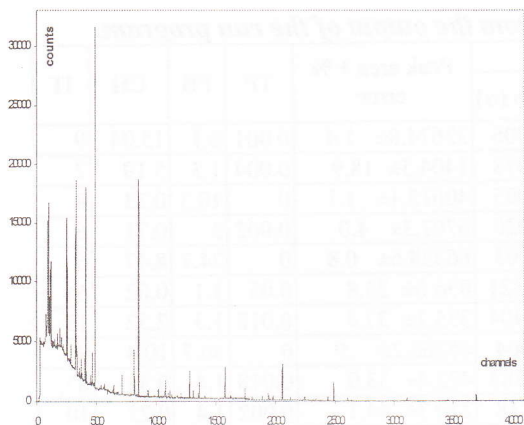


Fig.(3):The analyzed whole spectrum in the present study in the channel

### Conclusion

Throughout this study, it is assumed that every peak is distributed about a pulse height location corresponding to the energy of the primary  $\gamma$ -ray according to the normal probability (gaussian distribution). A computer program was designed to search for peaks automatically by locating adjacent channels such that the derivative in the left side of the peak is positive and negative for the right side of the peak. Then Marquardt method was followed by the program to fit the data point for getting most probable gaussian parameters. Finally, the used spectrum was resolved to it's components about (56) lines and the present results were in conformity with those of (N.D.S.).

In addition; and through the spectrum, about (nine) common  $\gamma$ -ray lines were identified by the program without resolving their components, but instead, their detection was done by comparing their relative intensities with the ones of (N.D.S.), so they had a great relative intensities in comparison with ones of (N.D.S.) because they have more than one nuclide contribution, as shown in Table (3).

**Table(3):Unresolved common lines in the used spectrum that are detected by comparing their relative intensities with those of (N.D.S.)**

No.	Nuclide	Energy ( $E_\gamma$ ) in keV	Relative intensities ( $I_\gamma$ ) in comparison	
			Present work	N.D.S.
1	$^{228}\text{Th}$	154.02	$4.83 \pm 0.019$	2.77
2	$^{231}\text{Th}$	185.92	$100 \pm 0.002$	100
3	$^{214}\text{Bi}$	241.89	$21.3 \pm 0.03$	20.13
4	$^{228}\text{Th}$	270.06	$32.68 \pm 0.1$	12.9
5	$^{208}\text{Pb}$	510.87	$30.61 \pm 0.032$	22.8
6	$^{228}\text{Th}$	727.43	$23.35 \pm 0.126$	2.4
7	$^{214}\text{Po}$	786.03	$3.43 \pm 0.216$	0.68
8	$^{228}\text{Th}$	839.45	$12.69 \pm 0.281$	3.53
9	$^{214}\text{Po}$	964.63	$3.65 \pm 0.273$	0.83

Table (2): A comparison between the present results and those obtained by (N.D.S.) [20,21,22,23,24]

No.	Peak Boundary	Nuclides	Energy (E <sub>γ</sub> ) in keV	Relative intensities (I <sub>γ</sub> ) in comparison	
				Present work	N.D.S.
1	171-177	B	128.92±.052	9.05±.006	9.2
2	192-199	A	143.74±.034	.....	18.95
3	219-227	A	163.19±.102	.....	8.69
4	280-285	A	205.43±.113	.....	8.69
5	285-292	B	209.25±.026	14.31±.014	14.6
6	327-332	E	238.58±.004	100.00±.002	100
7	356-361	C	258.83±.052	1.58±.037	1.485
8	382-388	F	277.38±.062	6.21±.046	6.6
9	406-413	C	295.20±.004	51.70±.003	51.72
10	413-419	E	299.5±.028	7.38±.034	7.99
11	453-460	B	333.07±.041	11.09±.044	11.1
12	467-473	B	338.31±.013	42.27±.013	42.3
13	484-494	C	351.92±.003	100.00±.003	100
14	541-546	D	389.03±.074	.81±.146	0.89
15	569-574	B	409.52±.051	7.59±.088	7.28
16	634-629	D	454.69±.158	1.09±.138	0.69
17	642-653	B	462.99±.036	16.78±.069	16.7
18	678-686	C	487.02±.106	1.46±.168	1.18
19	786-791	E	562.74±.203	3.65±.327	3.27
20	814-821	F	583.35±.009	86.09±.023	85.2
21	849-860	D	609.76±.003	100.00±.009	100
22	931-936	D	625.64±.042	3.54±.103	3.39
23	981-989	D	703.62±.138	1.07±.456	1.02
24	1009-1014	D	719.95±.113	1.05±.378	0.87
25	1059-1064	B	755.57±.072	4.09±.556	3.78
26	1074-1083	D	768.49±.019	10.71±.072	10.59
27	1113-1121	B	795.04±.025	16.17±.207	16.3
28	1127-1138	D	806.38±.055	3.17±.233	2.67
29	1205-1214	F	860.56±.041	12.74±.298	12.53
30	1269-1274	B	904.04±.123	3.56±.961	2.94
31	1277-1286	B	911.09±.013	100.00±.064	100
32	1309-1317	D	934.14±.029	6.67±.148	6.86
33	1360-1367	B	969.01±.016	60.57±.102	60.8
34	1572-1582	D	1120.29±.011	32.44±.076	32.62
35	1623-1630	D	1155.05±.064	3.98±.382	3.67
36	1740-1749	D	1238.14±.021	12.72±.188	12.84
37	1800-1809	D	1281.00±.068	3.21±.606	3.19
38	1937-1947	D	1377.72±.030	8.84±.292	8.72
39	1950-1956	D	1385.54±.142	1.65±.1029	1.68
40	1970-1980	D	1401.65±.063	3.36±.662	3.0
41	1980-1989	D	1408.07±.040	5.87±.405	5.37
42	2125-2132	D	1509.32±.047	4.77±.532	4.75
43	2174-2180	D	1543.64±.139	4.02±.2502	0.76
44	2236-2243	B	1588.37±.030	12.37±.1032	12.3
45	2280-2290	G	1620.61±.207	19.86±.2599	19.86
46	2296-2305	B	1630.58±.113	6.62±.1677	6.72
47	2339-2347	D	1661.39±.069	2.32±.1028	2.49
48	2434-2445	D	1729.56±.036	6.88±.507	6.6
49	2485-2494	D	1764.68±.015	34.37±.184	34.54
50	2590-2598	D	1838.40±.256	.91±.2.781	0.83
51	2603-2612	D	1847.65±.047	4.84±.712	4.6
52	2985-2995	D	2118.66±.072	2.70±.1.556	2.62
53	3106-3120	D	2204.33±.033	11.05±.648	10.83
54	3235-3242	D	2293.29±.261	1.00±.3.727	0.7
55	3451-3463	D	2448.03±.063	3.83±.1.478	3.37
56	3687-3701	F	2614.95±.025	100.00±.577	100

\* A: <sup>231</sup>Th, B: <sup>228</sup>Bi, D: <sup>214</sup>Po, E: <sup>212</sup>Bi, F: <sup>208</sup>Pb, G: <sup>212</sup>Po  
 ..... indicates those nuclides that their relative intensities have not found.

## References:

- [1] Blaaw, V. Osorio Fornondez. and W. estmeier, *Nucl.Inst. and Meth.* **A387 (1997) 410.**
- [2] A. H. Ahmad, *Sc. Thesis, Salahaddin Univ., Iraq, (1992).*
- [3] J. T. Routti and S. G. Prussin, *Nucl.Inst. and Meth.* **72(1969) 125.**
- [4] M. A. Hammed, P. W. Gray, A.H. Nabulsi and T. D. Mac Mahan, *Nucl.Inst. and Meth.* **A334(1993)543.**
- [5] O. M. Mustaffa and A. H. Ahmad, *ZANCO*, June **(1997)153.**
- [6] A. G. Smith and W. J. Vermeer, *Nucl.Inst. and Meth.* **A350(1994)314.**
- [7] D. C. Robinson, *Nucl.Inst. and Meth.* **78(1970)120.**
- [8] F. Beckely Smith, Jr. and David F. Shano, *Technometric*, Vol. 13, No. 19, 19 February **(1971)63.**
- [9] H. P. William, P. F. Brain, A. T. Saul and T.V. William, *Numerical Recipes*, **(New York, 1992).**
- [10] P. R. Bevington, *Data Reduction and Error Analysis for the Physical Sciences*, **(McGraw-Hill, New York, 1969).**
- [11] R. G. Helmer and C. G. M. McCullagh, *Nucl.Inst. and Meth.* **206(1983)477.**
- [12] H. Machner, *Nucl.Inst. and Meth.* **A258(1987)246.**
- [13] W. Gray Philips and W. Marlow Keith, *IEEE Transactions on Nuclear Science*, Vol. Ns-24, No. 1, **February (1977)154.**
- [14] S. J. Mills, *Nucl.Inst. and Meth.* **81(1970)217.**
- [15] I. A. Slavic, *Nucl.Inst. and Meth.* **112(1973)253.**
- [16] A. L. Connely and W. W. Black, *Nucl.Inst. and Meth.* **82(1970)141.**
- [17] B. Nyman, *Nucl.Inst. and Meth.* **108(1973)237-241.**
- [18] S. O. Hassan, *M. Sc. Thesis, Salahaddin Univ., Iraq, (1999).*
- [19] A. H. Fattah, *Sc. Thesis, Salahaddin Univ., Iraq, (2000).*
- [20] K. S. Toth, *Nuclear Data Sheets*, Vol. 21, No. 4, **August (1977).**
- [21] M. J. Martin, *Nuclear Data Sheets*, Vol. 27, No. 4, **August (1979).**
- [22] M. R. Schmorak, *Nuclear Data Sheets*, Vol. 40, No. 1, **September (1983).**
- [23] M. J. Martin, *Nuclear Data Sheets*, Vol. 47, No. 4, **April (1986).**
- [24] M. J. Martin, *Nuclear Data Sheets*, Vol. 4, No. 1, **September (1986).**
- [25] H. Siegert and H. Janssen., *Nucl.Inst. and Meth.* **A286(1990)415.**

